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ADDRESS OF THE RETIRING PRESIDENT OF THE SOCIETY, IN AWARDING THE BRUCE MEDAL TO DR. GEORGE WILLIAM HILL.

By CHARLES BURCKHALTER.

The eighth award of the Bruce Gold Medal of this Society has been made to Dr. GEORGE WILLIAM HILL.

To those having understanding of the statutes, and the method governing the bestowal of the medal, it goes without saying that it is always worthily bestowed. The statutes require that, for each medal awarded, eminent astronomers,—the directors of six of the greatest observatories in the world,—shall be asked each to nominate three astronomers worthy to receive the Bruce Medal "for distinguished services to astronomy." The six observatories have always been Harvard, Lick, and Yerkes in this country, and Berlin, Greenwich and Paris in the Old World. From the nominations thus made the Directors of this Society, by ballot, choose the Medalist.

The list of names of those whom we have thus honored, and in honoring them we have honored the Society, is an inspiring one—all household names in astronomy. The first award was to SIMON NEWCOMB, a native of Nova Scotia; the second to ARTHUR AUWERS, a German; the third to Sir DAVID GILL, an Englishman; the fourth to GIOVANNI VIRGINIO SCHIAPARELLI, an Italian; the fifth to Sir WILLIAM HUGGINS, an Englishman; the sixth to another German, HERMANN CARL VOGEL; the seventh to EDWARD C. PICKERING, the first native-born American to receive the medal; and the eighth to our present medalist, also an American. The Bruce Medal, like astronomy, knows nothing of boundaries.

In a paper in the *Popular Science Monthly* for October, 1908, Professor PICKERING calls attention to the fact that there are only six Americans who are members of three or more of the seven leading National Scientific Societies (England, France, United States, Russia, Germany, Austria and Italy) and of these six, three are astronomers; and I wish to add, of the three astronomers our medalist is one. Professor PICKERING also points out that every living Bruce Medalist belongs to three or more of these societies.

Although this is but the eighth award of the medal, so great is the care taken in its bestowal, and with such wisdom have the selections been made, that the Bruce Gold Medal is already recognized as one of the greatest honors that can be conferred upon an astronomer.

The statutes for the bestowal of the Bruce Medal require the President of the Society at this meeting to "Announce the award and the reasons for making it." Let me here say that no adequate review of Dr. HILL'S work can be comprised in this address; in his "Collected Mathematical Works" there are eighty-four memoirs, every one of which shows the master mind. And I wish at once, frankly, to confess my inability to give an opinion of the value of Dr. HILL'S services to astronomy, and in this, your President feels that he is but one of a large company. Only specialists in celestial mechanics are competent to appreciate and appraise the utility and importance of his monumental achievements; the opinions of those intellectual giants who are his colaborers, alone can reflect to us a true estimate of their value.

Professor NEWCOMB in his "Reminiscences of an Astronomer" says of Dr. HILL that "he will easily rank as the greatest master of mathematical astronomy during the last quarter of the nineteenth century." Wedded to his chosen field of labor, he worked, year after year, scarcely known to the public, patient, underpaid, his ability appreciated by only a few of his colleagues, but content that he could advance his beloved science. Speaking of this time, NEWCOMB says of him: "Here was perhaps the greatest living master in the highest and most difficult field of astronomy, winning world-wide recognition for his country in the science and receiving the salary of a department clerk."

The list of honors conferred upon our medalist by institutions of learning in this and foreign countries is a long one, among which I may mention the Damoiseau prize, of the Paris academy, the degree of Doctor of Laws, by Cambridge University, England, and the gold medal of the Royal Astronomical Society. Our own recognition is, of necessity, somewhat tardy, for he has been nominated again and again by eminent astronomers, for the honor of the Bruce Medal.

I have said that only an expert can rightfully weigh such work as that of our medalist. Such an one is Mr. HENRI POINCARÉ, who has written, in French, an introduction to the "Collected Mathematical Works of George William Hill." From this, the following abstract has been prepared by Dr. R. G. AITKEN, of the Lick Observatory:—

"Dr. HILL," says POINCARÉ, "is one of the most original figures in the American scientific world. Throughout all his works and his calculations he has remained a stranger to the feverish life that has troubled others; he has conducted his researches in isolation, formerly in the bureau of the Nautical Almanac, more recently at his quiet home in the Hudson Valley. This reserve, I was about to say this shyness, has been a happy circumstance for science, for it has permitted him to carry his ingenious and patient researches to their conclusion without suffering distractions from the constant accidents of the world outside."

Dr. HILL was born in New York, March 3, 1838. His father, an Englishman, came to America in 1820 at the age of eight; his mother, of an old Huguenot family, transmitted to him the traditions of the earliest colonists of America. He passed his infancy at the farm at West Nyack, about twenty-five miles from New York, which his father bought shortly after HILL's birth.

"He always loved his home, he returned to it as often as he could, and when he resigned from the Nautical Almanac Office he made his permanent home there, and there he pursues his studies in tranquility, avoiding, as far as possible, even journeys to New York."

At the age of seventeen he entered Rutger's college, New Brunswick, New Jersey, where his professor in mathematics was Dr. STRONG, a friend of BOWDITCH, the translator of "Laplace's Mécanique Céleste."

"Dr. STRONG was a man of traditions, one who praised the bygone times; for him EULER was the god of mathematics, and after EULER, decadence began; it is true that this is a god one may worship with profit. With rare exceptions Dr. STRONG'S library was pitilessly closed against all books of later date than 1840. Happily, excellent works on celestial mechanics were written prior to 1840, for example, those of LAPLACE, LAGRANGE, POISSON, and PONTÉCOULANT. These were the masters by whom HILL was introduced to the rudiments of the science."

Receiving his degree in 1859 he went to Harvard to continue his mathematical studies, but in the spring of 1861 joined the Nautical Almanac staff and spent the next thirty years of his life in that service. Those were the most fruitful years in point of scientific productions. At this time the Nautical Almanac Office was at Cambridge, Mass., under the direction of Professor RUNKLE, the founder of the *Mathematical Monthly,* in which prizes were proposed for the solution of mathematical problems. "One of the first articles published," says POINCARÉ, "revealed the hand of a master and easily gained the prize. It dealt with the functions of LAPLACE and the figure of the Earth. The author was Mr. HILL, who was just about to leave college."

HILL continued to make many contributions on various mathematical subjects, generally, however, connected in some way with celestial mechanics, to the *Mathematical Monthly,* and to the *Analyst* and similar journals.

While the Nautical Almanac Office remained in Cambridge under RUNKLE, HILL'S work for it was done at his home in West Nyack. But, in 1877, when NEWCOMB assumed charge, "he wished to undertake a colossal task, the reconstruction of the tables of all the planets; Mr. HILL'S part in this task was the most difficult; it was the theory of *Jupiter* and *Saturn*, with which he began to busy himself about 1872." He could not well carry on this work away from his chief and his colleagues, so was obliged to leave his home, a sacrifice readily made in view of the importance of the work. But he returned to his home when the bureau was moved to Washington.

In 1892 he resigned from the Nautical Almanac. For a short time thereafter he held a professorship in Columbia University, but he did not continue in this position long, and since then "has lived alone with his books and his souvenirs."

Inspection of the tables of contents in the four volumes of his "Collected Mathematical Works," published by the Carnegie Institution, of Washington, shows what a wide range his investigations took, in every department of celestial mechanics.

HILL's greatest work, according to all competent judges, consists in his researches on, and contributions to, the lunar theory. This account of it is based on POINCARÉ's introduction. He says it is the work to which HILL "devoted all the originality of his genius."

To comprehend HILL's work, POINCARÉ gives a brief account of the state of the theory when HILL began his work. Two great works on this subject had been completed at this time—HANSEN's and DELAUNAY's—each exhibiting the results of the highest sagacity and of extreme patience.

The methods adopted by these two investigators are radically different; HANSEN, who completed his work first, had in mind the purely utilitarian object of computing accurate lunar tables, hence he calculated numerical co-efficients directly and did not trouble himself to find analytical expressions. The tables in actual use to-day are based on HANSEN's calculations, and it is probable that the new theories, more scientific, more satisfactory in their spirit, will not give very different results. DELAUNAY, on the other hand, presented his inequalities in the form of algebraic formulas. "He gives us, then, not only the theory of the Moon, but the theory of any satellite that revolves, or that may revolve about any planet whatever. From this point of view he leaves HANSEN far in the rear. The method which has led him to this result constitutes the most important advance that has been made in celestial mechanics since LAPLACE. To-day, perfected and shortened, it has become an instrument that every one can use, and that has already rendered good service in every part of astronomy."

Unfortunately DELAUNAY's series converges only with excessive slowness and is therefore unsuited to numerical computations. HILL promptly mastered DELAUNAY's theory and made it the subject of various memoirs, but his own method is very different and very original.

DELAUNAY'S series depends on five constants, the eccentricities, the inclination, the solar parallax, and a quantity called *m*, which depends upon the mean motions. If we suppose the first four to become zero, we have a particular solution of the differential equations. This solution will be very much more simple than the general one since the greater part of the inequalities will disappear, the only remaining one being the one known as the "variation."

On the other hand this solution does not precisely represent the Moon's trajectory, but it will serve as a first approximation, since the neglected constants are in effect very small. The choice of *this* first approximation is far more advantageous than the Keplerian ellipse, since for this ellipse the perigee is fixed, whereas in the real orbit it is in motion. The differential equations are at the same time simplified, since, the eccentricity and parallax being zero, the Sun is supposed to describe a circumference of very great radius. Mr. HILL again simplified the equations by judiciously choosing his variables. He does not express them in polar coördinates, but in the rectangular, and this is a great step in advance. Again his variables are not referred to fixed axes, but to axes possessing a uniform rotation, equal to the mean angular motion of the Sun. This was a new simplification, since the time no longer figures explicitly in the equations. "But the most important advantage is the following: For an observer situated on those moving axes the Moon will appear to describe a closed curve, if the eccentricities, the inclination and the parallax are zero. As the differential equations are otherwise rigorous, *this is the first example of a periodic solution of the problem of three bodies* whose existence has been rigorously demonstrated. Recently periodic solutions have assumed capital importance in celestial mechanics. [POINCARÉ himself is one of the foremost investigators in this branch, if indeed he has an equal.] But our medalist was not content merely to demonstrate its existence; he studied this orbit (or these orbits) in detail, and determined point by point the closed trajectories and calculated the coördinates of these points to many decimals."

Not to follow POINCARÉ's discussion in too much detail, it may suffice here to say that in the further transformations and solutions of these equations, Mr. HILL showed as much daring as originality. His solution involved the consideration of *an infinite number of linear equations.* Not only did he not hesitate to consider these, but he also studied determinants of an infinite order, a thing never before attempted[1] and, happily, his daring was justified by his success.

If we compare his methods with those of DELAUNAY, we shall find that HILL's method by three approximations gave results for the constant of the motion of the lunar perigee correct certainly to thirteen decimals, whereas (to quote from HILL's own paper) "Although DELAUNAY has been at the great pains of computing eight terms of this series, they do not suffice to give correctly the first four significant figures [eight decimals] of the quantity sought. . . . As well as can be judged from induction it would be necessary to prolong the series, in powers of m, as far as m^{27}, in order to obtain an equally precise result."

This will suffice to illustrate the great advance made by HILL in the development of the lunar theory, for the method thus applied to the motion of the lunar perigee may also be applied to the motion of the node. The main difficulties are thus conquered, and the subsequent approximations are relatively easy. Nevertheless practical difficulties still remain, and the field is open for new improvements and new theories. It is unnecessary in this connection to do more than merely to refer to the work of another eminent investigator, Professor E. W. BROWN, who has developed methods of which much is expected.

The inequalities in the lunar motion so far discussed have been those due to the action of the Sun. They are the ones that would be produced if only the Sun, the Moon, and the Earth existed, and they were reduced to material points. But there are other inequalities in the lunar motion, produced in part by the direct action upon the Moon of the other planets, and in part, indirectly, by the disturbing action of these planets upon the motion of the Earth about the Sun. Further, the Earth is not spherical, and its equatorial protuberance exercises an influence upon the lunar motion.

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[1] At least not to Dr. HILL's knowledge at this time. It appears that ADAMS, of Neptunian fame, had been led by his researches to a determinant very nearly identical with the one discovered by HILL. POINCARÉ also refers to a memoir by M. KOTTERITZSCH in *Poggendorf's Annalen,* though he says the method there given had "nothing in common with that of the American Geometer."

In the planetary perturbations of the Moon's motion we may further distinguish between the secular variations and the periodic. Mr. HILL has studied successively the secular acceleration of the mean motion, that of the motion of the perigee, and the influence of the variations of the ecliptic. Nor has he neglected the others, and he has also given careful study to the effect of the oblateness of the Earth and discussed the results of the pendulum determinations of the force of gravity on the Earth's surface.

But the lunar theory did not absorb all of HILL's time or energy. Time and again he turned to investigations dealing with general problems, the general theory of planetary perturbations and special problems in this field. His largest piece of work of this kind is the one undertaken for the Nautical Almanac in connection with and as part of NEWCOMB's "colossal task," referred to above, of the complete discussion of the mutual perturbations of *Jupiter* and *Saturn*.

LAPLACE had entered upon this theory which presents very great difficulties because of what is termed "the great inequality," but his evaluation of the terms of the second order was but a rough approximation. HANSEN was more fortunate, and so arranged his calculations that he was able to estimate the importance of terms he neglected, but he carried out his work completely only in the case of *Saturn*. For *Jupiter* he was content to stop with terms of the first order. We may pass over other memoirs as relatively unimportant until we come to that of LE VERRIER, published in 1876. His theory is indeed complete, but his formulæ are entirely literal, and so arranged that the co-efficients of the inequalities are expressed in terms of corrections to all of the osculating elliptic elements. In practice this leads to excessively long computations and there are apparently some outstanding errors in the formulæ; besides, the tables are not convenient to use. But HILL began his investigations in 1872, before LE VERRIER's results were published and at a time when it was uncertain when they would be published. The tables then in actual use were BOUVARD'S, and these were quite inadequate to meet the needs of astronomers. Having a purely practical end in view, the construction of accurate tables in the shortest possible time, HILL did not seek to develop a new method of investigation. He adopted HANSEN'S, but with modifications that greatly simplified it. So great an amount of computing was involved in this research that Mr. HILL devoted to it seven and a half years of time, being relieved during this time of all routine duties in the Nautical Almanac Office and having the services of an assistant to check by duplicate computations all the more important calculations. The result of this long, and in many ways necessarily tedious work is a splendid volume of theoretical researches, and two volumes of accurate tables, one of the motion of *Jupiter,* the other of the motion of *Saturn.*

The recent progress of celestial mechanics has received and continues to receive Mr. HILL'S constant attention. HANSEN'S and DELAUNAY'S methods, GYLDÉN'S intermediate orbit, the recent developments in periodic solutions of the three-body problem [as presented by POINCARÉ, DARWIN and others], have all been assimilated, and many of them analyzed and discussed in some of his published memoirs.

In his concluding sentence POINCARÉ says: "There is therefore not a single part of celestial mechanics to which he is a stranger, but his chief work, that which will make his name immortal, is his theory of the Moon. It is there that he is not only an able artist, a careful investigator, but an *inventor,* original and profound. I do not wish to say that the methods which he created are applicable only to the Moon. I am well persuaded to the contrary; I think that those who are engaged in study of the minor planets will be astonished at the relief from difficulties they will experience on the day when, having fathomed their spirit, they apply them to this new object. But at present it is for the Moon that they have proved themselves. When they enter upon a wider domain, we ought not to forget that it is to Mr. HILL that we owe so precious an instrument."

We have earnestly hoped that our medalist would be with us this evening, but in a letter to Dr. HALE he said he did not feel strong enough to undertake the journey; therefore, in his absence, I hand to you, Mr. Secretary, for transmission to Dr. Hill, this medal, the highest honor this Society can possibly confer upon an astronomer, and with it I ask you to send the greetings and best wishes of the Astronomical Society of the Pacific, and that we trust the evening of his life may be a long and happy one.

March 27, 1908.

THE LAWS OF COSMICAL EVOLUTION AND THE EXTENSION OF THE SOLAR SYSTEM BEYOND NEPTUNE.

By T. J. J. See.

(Abstract of address delivered January 30, 1909.)

It has long been considered somewhat of a reproach to astronomy that the processes of cosmogony have remained so obscure that definite laws could not be established regarding even the mode of formation of the solar system, while still less was known about the laws for the development of other systems in space. In view of the great progress of the physical sciences since the time of Laplace, one is compelled to admit that this criticism of the oldest and most exact of the physical sciences is not wholly unjust and without a certain foundation. Not only has the failure of researches in cosmogony affected astronomy adversely, but it has also narrowed the field of effort in several of the related sciences. This should not, however, occasion surprise among those who study the history of the physical sciences. For as cosmogony depends on the other sciences for its fundamental data, any circumstance which has affected them adversely would also retard the development of cosmogony itself, and *vice versa*.

In addition to the natural difficulties inherent in the development of a complex and dependent science like cosmogony, another has arisen from the demoralization of spirits due to the failures of previous investigators. Those who have labored for years without gaining any satisfactory light on the subject may easily convince themselves that there are no definite laws